Forest Reflectance and Transmittance
FRT User Guide

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Contents

Abstract

A directional multispectral forest reflectance model has been developed in the group of vegetation remote sensing at Tartu Observatory, Estonia. The early version of the forest reflectance model by Nilson (1991) has been extensively modified. The modified leaf optics models PROSPECT by Jacquemoud et al. (1996) and LIBERTY by Dawson et al. (1998), atmosphere radiative transfer model 6S by Vermote et al. (1994, 1997), and homogeneous two-layer canopy reflectance model ACRM by Kuusk (2001) have been incorporated into the model. The new model works in the spectral region 400-2400 nm with the same set of input parameters, the spectral resolution is 1 nm. Any Sun and view directions are allowed. The following manual presents the 2024 version of the model, the algorithms of which are those of the FRT13, but the computer code is modified so that the model is using the format of input files of the FRT23.

1 Introduction

The transfer of solar radiation within forest stands is a rather complex process. We need models to understand how the reflected signal is formed and which are its most important driving factors. In addition, to create a satellite or aerial imagery-based forest management system, forest reflectance models capable of acting as an interface between the images and forestry databases are required. These models should be able to make maximum use of the forestry data contained in the database and allow to simulate the optical images, e.g. in terms of standwise ground-level reflectance factors. Originally, the forest reflectance model described in Nilson and Peterson (1991) has been derived just from these starting points. The previous version of the model needed several improvements. First of all, to make use of multiangular remote sensing data, the model should be modified into a multiangular version. Second, a multispectral version of the model is required to study the relations between leaf biochemical and high spectral resolution reflectance data. Several improvements were also needed to create a more user-friendly version of the model and to introduce some changes in the calculation algorithms. For these purposes, a considerable modification of the original model was undertaken.

2 General layout of the model

The forest reflectance model may be classified as a hybrid-type model, including the properties both geometrical and radiative transfer equation-based models. Tree crown envelopes are modeled as ellipsoids of rotation or cones in the upper and cylinders in the lower part (Fig. 1). Leaves and branches are uniformly distributed in the crown.

Several tree classes of different size and/or species are possible (Fig. 1). Within each class, trees are considered identical.

A homogeneous layer of vegetation is present on the ground surface.

The radiances of the forested scene components – tree leaves/needles, branches and stems, ground vegetation, and soil – are estimated with the help of geometrical and radiative transfer concepts. Special attention is paid to the adequate modeling of single scattering reflectance components, whereas reflectance caused by multiple scattering of radiation in the canopy is more roughly modeled.

The directional spectral reflectance of a forest stand in the given direction r_2 is calculated as a sum of the single scattering reflectance $\rho_I(r_1, r_2)$ and diffuse reflectance $\rho_D(r_2)$,

$$\rho(r_1, r_2) = \frac{I_\lambda}{Q_\lambda}\rho_I(r_1, r_2) + \rho_D(r_1, r_2)\,, \tag{1}$$

where $I_\lambda = I_\lambda(\theta_1)\, cos(\theta_1)$ is direct down-welling flux, and $Q_\lambda = I_\lambda + D_\lambda$ is the total down-welling flux, D_λ is diffuse downwelling flux, r_1 and r_2 are unit vectors in the Sun and view direction, respectively, θ_1 is the Sun zenith angle.

The single scattering reflectance factor $\rho_I(r_1, r_2)$ accounts for the single scattering from tree layer foliage and stems $\rho^1_{CR}(r_1, r_2)$, and single scattering from ground vegetation $\rho^1_{GR}(r_1, r_2)$,

$$\rho_I(r_1, r_2) = \rho^1_{CR}(r_1, r_2) + \rho^1_{GR}(r_1, r_2). \tag{2}$$

Diffuse reflectance $\rho_D(r_1, r_2)$ accounts both for the multiple scattering of radiation and for the diffuse radiance of scattered/reflected sky radiation D_λ.

The model works in the optical domain of radiation, 400-2400 nm, spectral resolution is 1 nm.

3 Model components

3.1 Single scattering in tree crowns

The first-order reflectance component $\rho^1_{CR}(r_1, r_2)$ is calculated separately for all tree classes,

$$\begin{aligned}
\rho^1_{CR}(r_1, r_2) &= \sum_{j=1}^{m} \rho^1_{CRj}, \\
\rho^1_{CRj} &= \lambda_j \iiint\limits_{V_j} u_j\, \Gamma_j(r_1, r_2)\, p_{00j}(x, y, z; r_1, r_2)\, dxdydz / \cos\theta_1
\end{aligned} \tag{3}$$

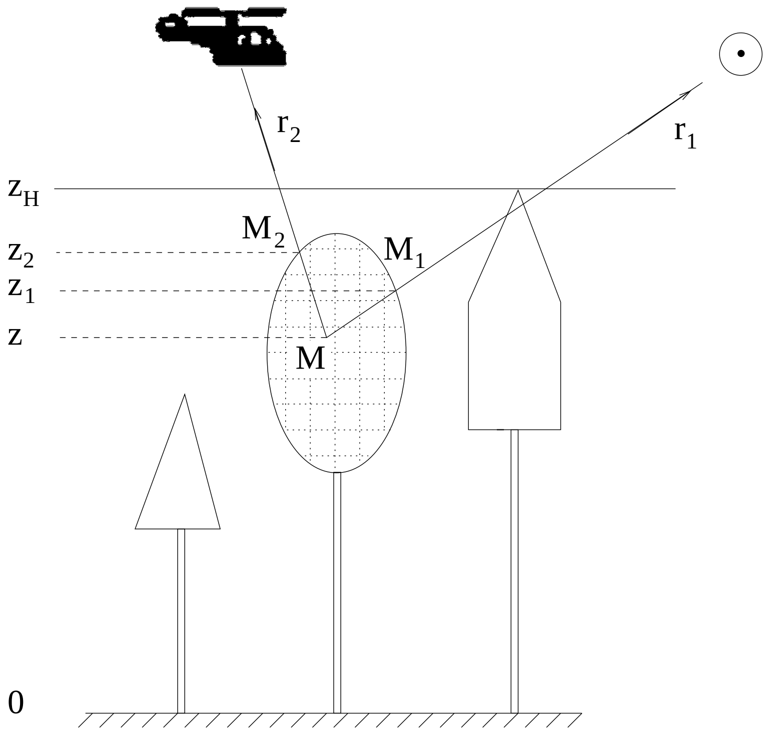

Figure 1: Deriving the first-order scattering component.

Here λ_j is the number of trees of the j th class per unit ground area, $u_j = u_j(x, y, z)$ is the foliage area volume density within a tree crown, $\Gamma_j(r_1, r_2)$ is the scattering (area) phase function of the canopy medium, $p_{00j}()$ is the bidirectional gap probability of two simultaneous free lines-of-sight in directions r_1 and r_2 from the point $M = (x, y, z)$ within a crown of the j th tree class (Fig. 1), V_j is the spatial region corresponding to the crown envelope. Integral (3) is calculated numerically using 3D quadrature.

The scattering phase function $\Gamma_j(r_1, r_2)$ in formula (3) is the sum of diffuse $\Gamma_{j,D}(r_1, r_2)$ and specular $\Gamma_{j,sp}(r_1, r_2)$ scattering,

$$\Gamma_j(r_1, r_2) = \Gamma_{j,D}(r_1, r_2) + \Gamma_{j,sp}(r_1, r_2). \tag{4}$$

Leaves of broadleaf species are supposed to be bi-Lambertian, foliage element reflection ρ_{Lj} and transmission τ_{Lj} coefficients are calculated with the PROSPECT submodel (Jacquemoud and Baret, 1990). Leaf refractive index n_{Lj} is a given tabulated function of wavelength. Foliage orientation is described by the two-parameter elliptical leaf angle distribution (LAD) (Kuusk, 1995a),

$$g_L(\theta_L) = B_g / \sqrt{1 - \epsilon^2 \, \cos^2(\theta_L - \theta_m)}, \tag{5}$$

where θ_L is leaf inclination, θ_m is the modal leaf inclination, and ϵ is the eccentricity of the LAD which determines the shape of the LAD, B_g is a normalizing factor. As the sensitivity range on the LAD eccentricity is very close to the limit value $\epsilon = 1$, the parameter $e_L = -\log(1-\epsilon)$ is used as the input parameter of FRT.

On these assumptions, the scattering phase function $\Gamma_{j,D}(r_1, r_2)$ in Eq. (4) may be calculated by analytical formulas in case of a few exceptional LAD (spherical, horizontal, vertical LAD, or fixed leaf angle) (Nilson, 1991), or by approximation formulae in case of elliptical LAD (Kuusk, 1995a).

For conifer species the asymmetric Henyey-Greenstein phase function is used (Lenoble, 1977),

$$\Gamma_{HG}(\gamma) = \frac{1-g^2}{\sqrt{(1+g^2-2\,g\,\cos(\gamma))^3}}, \tag{6}$$

where g is the asymmetry parameter, $-1 \leq g \leq 1, \gamma$ is the angle between sun and view directions.

Optical parameters are averaged over all foliage elements (leaves, branches) according to their share in the total foliage area.

The bidirectional gap probability p_{00j} is defined as a product of two independent probabilities

$$p_{00j} = p_1\,p_2 \tag{7}$$

p_1 being the within-crown level bidirectional gap probability and p_2 that of the between-crown level. In calculations of the bidirectional gap probability p_1, results from (Kuusk, 1991) for the crown of a single tree are applied. The mutual shading of needles in shoots and the characteristic linear dimension of foliage elements l_{sh} are accounted for.

The between-crown gap probability, p_2, in Eq. (7) stands for the parts of the lines-of-sight that lie outside the crown of interest, i.e. from the point $M_1(x_1, y_1, z_1)$ until the upper boundary of the forest canopy in the solar direction and from $M_2(x_2, y_2, z_2)$ in the view direction (Fig. 1). Based on (Nilson, 1977) it is calculated as follows:

$$p_2 = a_s(z_1, \theta_1)\,a_s(z_2, \theta_2)\,C_{HS2}(z_1, z_2, l_{12}, r_1, r_2)\,, \tag{8}$$

where $a_s(z, \theta)$ is the average proportion of gaps in the forest canopy at the height z in the direction θ, and C_{HS2} is the hot-spot correction factor for between-crown shading,

$$C_{HS2}(z_1, z_2, l_{12}, r_1, r_2) = \exp\left[\sum_j \lambda_j c_j S_{cj}(z_1, z_2, l_{12}, r_1, r_2)\,p_{0j}\right]\,, \tag{9}$$

$S_{cj}(z_1, z_2, l_{12}, r_1, r_2)$ is the area of the common part of the j th class crown envelope projections in solar and view directions, corresponding to the heights z_1 and z_2 and the horizontal distance l_{12}; p_{0j} is the joint probability of gap occurrence within a single j th class tree crown when viewed simultaneously from a point at the height z_1 in the solar direction r_1 and from another point at the height z_2 in the view direction r_2, horizontal distance of the points being l_{12}. The parameter c_j is introduced to account for the deviations in the tree distribution pattern from the Poisson distribution, see Eq. (17).

The gap probability $a_s(z,\theta)$ is calculated on the assumption of the binomial distribution of trees (Nilson, 1977),

$$a_s(z,\theta_r) = \exp\left\{-\sum_j \lambda_j [b_{1j}(z,\theta_r) S_{crown,j}(z,\theta_r) + S_{trunk,j}(z,\theta_r)]\right\}, \tag{10}$$

where $b_{1j}(z,\theta_r) = \ln[1-(1-a_{1j}(z,\theta_r))(1-c_j)]/(1-c_j)$, $S_{crown,j}(z,\theta_r)$ is the area of crown envelope projection for class j at the level z, and $S_{trunk,j}(z,\theta_r)$ is the area of trunk projection for class j at the level z, $a_{1,j}(z,\theta_r)$ is the gap probability in crowns of the tree class j in the direction θ_r at the level z, θ_r is the polar angle of the view vector $r_i, i = 1,2$. The area of trunk projection $S_{trunk,j}(z,\theta_r)$ is calculated using trunk tapering curves by Ozolins (1988). The function $a_{1j}(z,\theta_r)$ is shown in Eq. (11),

$$a_{1j}(z,\theta_r) = exp(-u_j\, G(\theta_r)\frac{V_j(z)}{S_{crown,j}(z,\theta_r)\cos(\theta_r)}), \tag{11}$$

$V_j(z)$ is the volume of the tree crown above the level z in the tree class j, $G(\theta_r)$ is the Ross-Nilson geometry function. In case of spherical LAD $G(\theta_r) = 1/2$, for other LAD-s the approximation by Kuusk (1995a) are used. As the crown envelopes are supposed to be surfaces of revolution, the between-crown gap probability $a_s(z,\theta_r)$ does not depend on the azimuth. Grouping and/or regularity of the stand is described by a grouping parameter c_j in every tree class, $c_j < 1$, $c_j = 1$, and $c_j > 1$ correspond to a regular, random, and clumped pattern of trees in class j, respectively. As the stem coverage (basal area) is very small, unlike the crowns, the stem position pattern is supposed to be random.

In Eq. (10), the expression $\lambda_j[S_{crown,j}(z,\theta_r) + S_{trunk,j}(z,\theta_r)]$ stands for the mean coverage of ground by the shadows cast by crown envelopes and trunks from tree class j, if the direction of sunrays coincide with the view direction θ_r. It is the effective coverage that should appear in the exponent of Eq. (10). The mean coverage should be diminished, because the tree crowns are supposed to be semi-transparent, and modified to account for the tree distribution pattern effect. The two effects of single-crown transparency and of the tree distribution pattern on the between-crown canopy gap fraction are introduced by the parameter $b_{1j}(z,\theta_r)$. Note that $b_{1j}(z,\theta_r) = 1 - a_{1,j}(z,\theta_r)$, if $c_j = 1$.

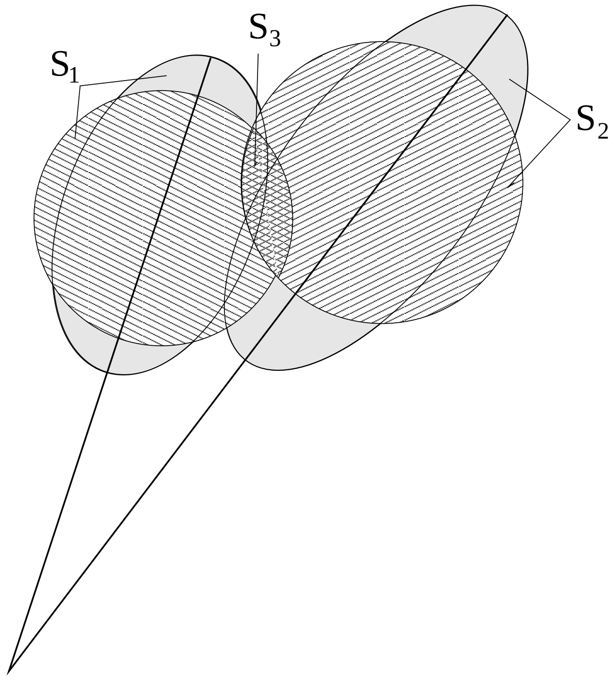

Figure 2: Calculation of the overlapping of crown projections.

The overlapping of crown projections in Sun and view directions $S_{cj}()$, which is needed for the calculation of between-crown level bidirectional gap probabilities, is estimated approximately. It is calculated so that the crown projections S_1 and S_2 in Sun and view directions, respectively, are substituted with circles of the same area. Centers of the circles are halfway between the projections of the base and the top of a crown, see Fig. 2. The estimated overlapping area S_3 in Fig. 2 may be biased to some extent. Depending on the Sun and view angles, the relative azimuth between Sun and view directions, and the tree height and the crown size, both over- and underestimation of the overlapping area S_3 are possible.

3.2 Single scattering on ground vegetation

The two-layer homogeneous canopy reflectance model ACRM by Kuusk (2001) is applied for the calculation of the bidirectional reflectance of ground vegetation. Input parameters of the ACRM are the leaf area index (LAI), leaf size, two leaf angle distribution parameters, the set of biophysical parameters (PRSOPECT parameters) for two layers of ground vegetation, and weights of Price's functions for the calculation of the soil reflectance spectrum. The probability of seeing sunlit ground vegetation is calculated as the p_2 in Eq. (8) for the ground surface, $z_1 = z_2 = l_{12} = 0$.

3.3 Diffuse fluxes

Diffuse fluxes of multiple scattering and of diffuse sky radiation are considered in four flux approximation like in the SAIL model (Verhoef, 1984) and in the ACRM model (Kuusk, 2001). Four differential equations define four fluxes: vertical fluxes up E_+ and down E_-, a direct solar flux E_s, and a flux associated with the radiance in the direction of observation E_o,

$$\begin{aligned} dE_+/dz &= -au_L E_+ + \sigma u_L E_- + s' u_L E_s \\ dE_-/dz &= -\sigma u_L E_+ + au_L E_- - su_L E_s \\ dE_s/dz &= ku_L E_s \\ dE_o/dz &= vu_L E_- + uu_L E_+ - Ku_L E_o \end{aligned} \tag{12}$$

The SAIL coefficients a, σ, s', s, k, v, u, and K are expressed using the G-function and leaf reflection and transmission coefficients ρ_L and τ_L. Equations (12) can be solved analytically, the general solutions for E_+, E_- and E_s are given, e.g. in (Bunnik, 1978).

The diffuse component of reflectance ρ_d is a sum of two components, related to tree layer and to ground vegetation, ρ_d^{trees} and ρ_d^{gr}, respectively,

$$\rho_d = \rho_d^{\text{trees}} + \rho_d^{\text{gr}}, \tag{13}$$

where

$$\begin{aligned} \rho_d^{\text{trees}} &= \text{SQ}\, r_{so} + (1-\text{SQ})\, r_{do} + \\ &+ [\text{SQ}\,(p_1\, r_{sd}^{\text{gr}} + t_{sd}\, r_{dd}^{\text{gr}}) + (1-\text{SQ})\, t_{dd}\, r_{dd}^{\text{gr}}]\, t_{do} \,/\, (1 - r_{dd}\, r_{dd}^{\text{gr}}) \end{aligned} \tag{14}$$

and

$$\rho_d^{\text{gr}} = [\text{SQ}(p_1\, r_{sd}^{\text{gr}}\, r_{dd} + t_{sd}) + (1-\text{SQ})\, t_{dd}] r_{ds}^{\text{gr}}\, p_2 \,/\, (1 - r_{dd}\, r_{dd}^{\text{gr}}). \tag{15}$$

Here $\text{SQ} = I_\lambda / Q_\lambda$, $p_i = p(r_i)$ is the gap probability in direction r_i, r_{sd}^{gr}, r_{ds}^{gr} and r_{dd}^{gr} are the directional-hemispherical, hemispherical-directional, and hemispherical-hemispherical reflectance of ground vegetation, respectively. The ground vegetation reflectances r_{sd}^{gr}, r_{ds}^{gr}, and r_{dd}^{gr} are calculated by integrating the ACRM model over hemisphere by view, incident, and both directions, respectively.

The scattering operators of the tree layer $r_{so}, r_{do}, t_{do}, t_{sd},$ and t_{dd} are defined in Table 1 where $D_\lambda = Q_\lambda - I_\lambda$.

When calculating diffuse fluxes, the plant material is supposed to be distributed homogeneously in the horizontal, no layers, no trees, no branches, no shoots, and driving parameters are determined as averages approximating the behavior of the canopy in bulk. The effective foliage area

Table 1: Scattering operators of the tree layer

Definition	Boundary conditions			
$r_{dd} = E_+(0)/E_-(0)$	$E_s(0) = 0,$	$E_+(-1) = 0,$	$E_-(0) = D_\lambda$	
$t_{dd} = E_-(-1)/E_-(0)$	$E_s(0) = 0,$	$E_+(-1) = 0,$	$E_-(0) = D_\lambda$	
$r_{sd} = E_+(0)/E_s(0)$	$E_s(0) = I_\lambda,$	$E_+(-1) = 0,$	$E_-(0) = 0$	
$t_{sd} = E_-(-1)/E_s(0)$	$E_s(0) = I_\lambda,$	$E_+(-1) = 0,$	$E_-(0) = 0$	
$r_{do} = E_o(0)/E_-(0)$	$E_s(0) = 0,$	$E_+(-1) = 0,$	$E_-(0) = D_\lambda$	
$t_{do} = E_o^-(-1)/E_-(0)$	$E_s(0) = 0,$	$E_+(-1) = 0,$	$E_-(0) = D_\lambda,$	$E_o^-(0) = 0$
$r_{so} = E_o(0)/E_s(0)$	$E_s(0) = I_\lambda,$	$E_+(-1) = 0,$	$E_-(0) = 0,$	$E_o(-1) = 0$

index value LAI_{eff} is used in the calculations of diffuse fluxes. LAI_{eff} is calculated from the gap probability in a given direction, it depends on the G-function of foliage and on the tree distribution pattern (clumping/regularity). As the G-function is almost invariant relative to leaf orientation at zenith angle 40° (Ross and Nilson, 1968), the effective LAI is calculated from the gap fraction at $\theta_0 = 40°$,

$$LAI_{eff} = \frac{\sum_j (\kappa_{cl,j} LAI_j + BAI_j)}{\Omega_E}, \tag{16}$$

where

$$\begin{aligned} \Omega_E &= \frac{\sum_j (\kappa_{cl,j} LAI_j + BAI_j)}{\cos\theta_0 \sum_j \lambda_j S_{crown,j}(\theta_0) c_j(\theta_0)}, \\ c_j(\theta_0) &= \frac{-\ln\left(1 - (1 - a_{1j}(\theta_0))\,(1 - GI_j)\right)}{1 - GI_j}. \end{aligned} \tag{17}$$

Here $\kappa_{cl,j}$ is the clumping coefficient of leaves/needles in a shoot of the tree class j, BAI_j is the branch area index, θ_1 is the Sun zenith angle, and $a_{1j}(\theta_1)$ is the gap probability in the Sun direction in crowns of the tree class j, GI_j is the Fisher's grouping index - the relative variance of the number of trees in the area $S_{crown,j}(\theta)$. The effective value of the foliage area index $LAI_{eff}^{(mult)}$ is calculated from the assumption that the gap fraction in the direction of sunrays as calculated by means of Eq. (10), and the modified exponential formula, as proposed in Chen and Cihlar (1996), should be equal. Thus, Ω_E could be interpreted as the 'clumping index caused by structures larger than a shoot'.

3.4 Leaf optics

Leaf optics models PROSPECT (Jacquemoud and Baret, 1990) or LIBERTY (Dawson et al., 1998) can be used for the calculation of leaf reflectance and transmittance in tree crowns. There is no option of using the LIBERTY model for the ground vegetation. Both these models are modified so that the number of leaf constituents and names of files of their extinction spectra are listed in the input file. Extinction spectra of the models PROSPECT2 (Jacquemoud et al., 1996), PROSPECT3 (Fourty and Baret, 1998), and LIBERTY (Dawson et al., 1998) are available. The structure parameter of a single leaf in the PROSPECT model N is corrected to an effective value N_{eff} in order to account for the overlapping of leaves/needles in a shoot,

$$N_{eff} = N/\kappa_{cl}\,. \tag{18}$$

If compared with the PROSPECT model, the LIBERTY model has two additional parameters: average internal cell diameter and intercellular air space determinant (Dawson et al., 1998).

In the forest model input, the biochemical parameters are expressed as a fraction of the dry matter of leaves/needles. Using the described set of biophysical parameters, the whole spectrum of leaf reflectance and transmittance in the spectral range 400-2400 nm is calculated with the spectral resolution of 1 nm.

No good optical model for branch and trunk bark reflectance is available so far. Therefore, reflectance spectra of branch and trunk reflectance for every tree class are tabulated in separate input files.

3.5 Sky radiation

The wavelength-dependent relative share of direct and diffuse flux in incoming radiation is needed, Eq. (1). The atmospheric radiative transfer model 6S by Vermote et al. (1997) is involved for the calculation of incident radiation fluxes. Input parameters of the 6S model, which are needed for the calculation of down-welling fluxes, are the percentage of four main aerosol components (dust-like, oceanic, water-soluble, and soot), and horizontal visibility or aerosol optical thickness at 550 nm τ_{aer}^{550}. The calculation of hemispherical-directional forest reflectance for sky radiation ρ_D is simplified. Instead of double integration over the hemisphere for incident directions, integration is performed in the perpendicular plane ($\varphi = 90°$) only,

$$\rho_D(r_2) = \frac{\int_{2\pi} d(r_1)\rho_I(r_1, r_2)\mu_1 dr_1}{D_\lambda} \approx \frac{\int_0^{\pi/2} d(\theta_1, \varphi = \pi/2)\rho_I(\theta_1, \theta_2, \varphi = \pi/2)\mu_1 d\theta_1}{D_\lambda}, \tag{19}$$

where $d(r_1)$ is the sky radiance in the direction $r_1 = (\theta_1, \varphi_1)$, $\mu_1 = \cos\theta_1$, and $D_\lambda = \int_{2\pi} d(r_1)\mu_1 dr_1$ is the diffuse down-welling flux from the sky.

4 Transmittance of a forest canopy

The same algorithms can be used for the calculation of downward radiances and fluxes under a forest canopy. The relative downward radiance in direction r_2 Sun being in direction r_1 is presented as the sum of three components:

$$t(r_1, r_2) = t^1_{CR}(r_1, r_2) + t_{sky}(r_1, r_2) + t^M_{CR}(r_1, r_2)\,. \tag{20}$$

Here the downward radiance $t(r_1, r_2)$ is normalized as reflectance in Eqs (2, 1), $t^1_{CR}(r_1, r_2)$ is the radiance of single scattering from tree crowns, $t_{sky}(r_1, r_2)$ is the sky radiance, and $t^M_{CR}(r_1, r_2)$ is the radiance of multiple scattering on crowns. In the model the sky radiance $t_{sky}()$ depends only on the Sun zenith angle θ_1.

Total transmittance of the tree layer $t_Q(r_1, r_2)$ is calculated as a ratio of the downward flux below the tree canopy to the incoming total flux Q_λ,

$$t_Q(r_1) = \frac{I}{Q_\lambda}\left(t^I_{CR}(r_1) + a_s(0, \theta_1)\right) + \frac{D_\lambda}{Q_\lambda}\int_{2\pi}\left(a_s(0, r_2) + t^I_{CR}(r_2)\right)\,\cos(\theta_2)\,dr_2\,, \tag{21}$$

and diffuse transmittance of the tree layer $t_D(r_1)$ is calculated as a ratio of the downward flux below the canopy (direct sunrays blocked) to the incoming diffuse flux D_λ,

$$t_D(r_1) = \int_{2\pi}\left(a_s(0, r_2)\cos(\theta_2) + t^I_{CR}(r_2)\right)\,\cos(\theta_2)\,dr_2 + \frac{I_\lambda}{D_\lambda}t^I_{CR}(r_1)\,. \tag{22}$$

Here $t^I_{CR}(r)$ is the scattering operator $I_\lambda(r) \rightarrow$ (downward scattered flux) for tree crowns.

5 Inversion of the model

Inversion of the model can be performed similar to Goel and Strebel (1983) or Kuusk (1991): a merit function is built, which has its minimum value when the best fit of measured and calculated reflectance data is reached. This way the complicated task of the solution of an array of non-linear equations for the estimation of model parameters is reduced to a more simple problem of the search of an extremum of a multidimensional function. In the merit function constraints are used in order to avoid the non-physical values of input parameters, and uncertainties of reflectance data and an expert estimate of parameter values are accounted for,

$$F(X) = \sum_{j=1}^{m}\left(\frac{\rho^*_j - \rho_j}{\epsilon_j}\right)^2 + \sum_{i=1}^{n}\left[(x_i - x_{i,b})^4\,w_i^2 + \left(\frac{x_i - x_{e,i}}{dx_i}\right)^2\right]\,. \tag{23}$$

Here $X = (x_1, x_2, ..., x_n)$ is the vector of model input parameters, m is the number of the measured reflectance values ρ_j^*, ρ_j is the model reflectance value, ϵ_j is the error of the measured reflectance value ρ_j^*, x_i is a model parameter and $x_{i,b}$ its value on the boundary of the given region; w_i is a weight, $w_i = 0$ in the given region $x_i \in [x_{i,min}, x_{i,max}]$ and $w_i =$ const else, $x_{e,i}$ is the expert estimate of the parameter x_i, and dx_i is a tolerance for the parameter x_i which controls the sensitivity of the merit function on the expert estimate.

There is an option to use only absolute differences $(\rho_j^* - \rho_j)^2$ in the merit function.

In the inversion, the redundancy of data can be effectively used, i.e. the number of reflectance values inverted may be more than the number of model parameters subject to estimation. Anyway, as the number of model parameters is large, most of the model parameters should be fixed at 'best guess' values, and only a few parameters can be estimated simultaneously. Only the parameters of the first tree class can be estimated in the inversion.

6 Conclusion

The model can be used for the interpretation of multispectral and/or multiangular remote sensing data in the wide range of Sun and view angles in the whole optical domain 400-2400 nm. The proposed version of the model seems to be a good tool for different sensitivity analyses, e.g. an analysis of the dependence of BRDF, in particular near the hot spot, on the stand structural variables at different structural levels and on optical parameters of the canopy and understorey can be made.

The same computer code can be used both for direct and inversion modeling.

The model is coded in Fortran. The computational aspects of the model are detailed in the following appendices:

- General description of the computer code
- Example of inputs and outputs
- Complete description of the subroutines

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Appendix

A General description of the computer code

A rough flowchart of the computer code is in Fig. 1, and the full call-tree in Fig. 2.

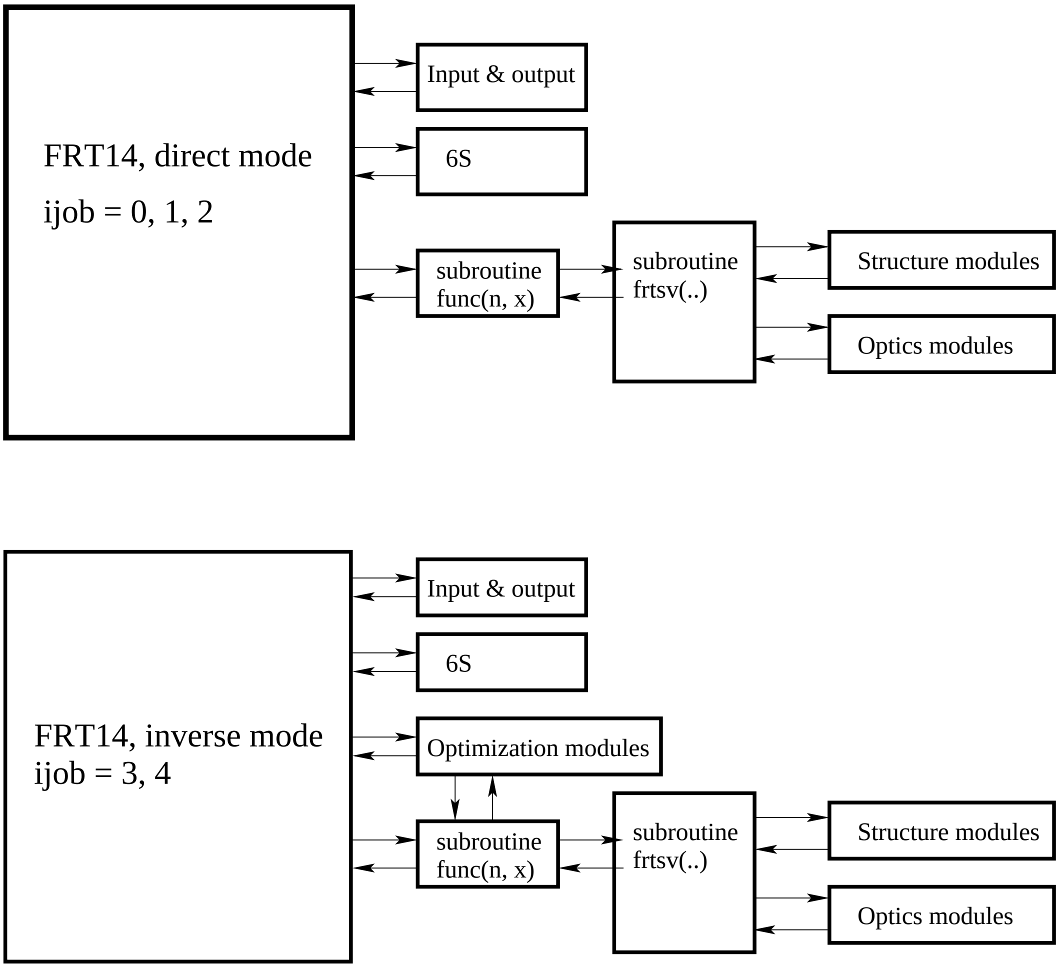

Figure 1: Flowchart of the computer code.

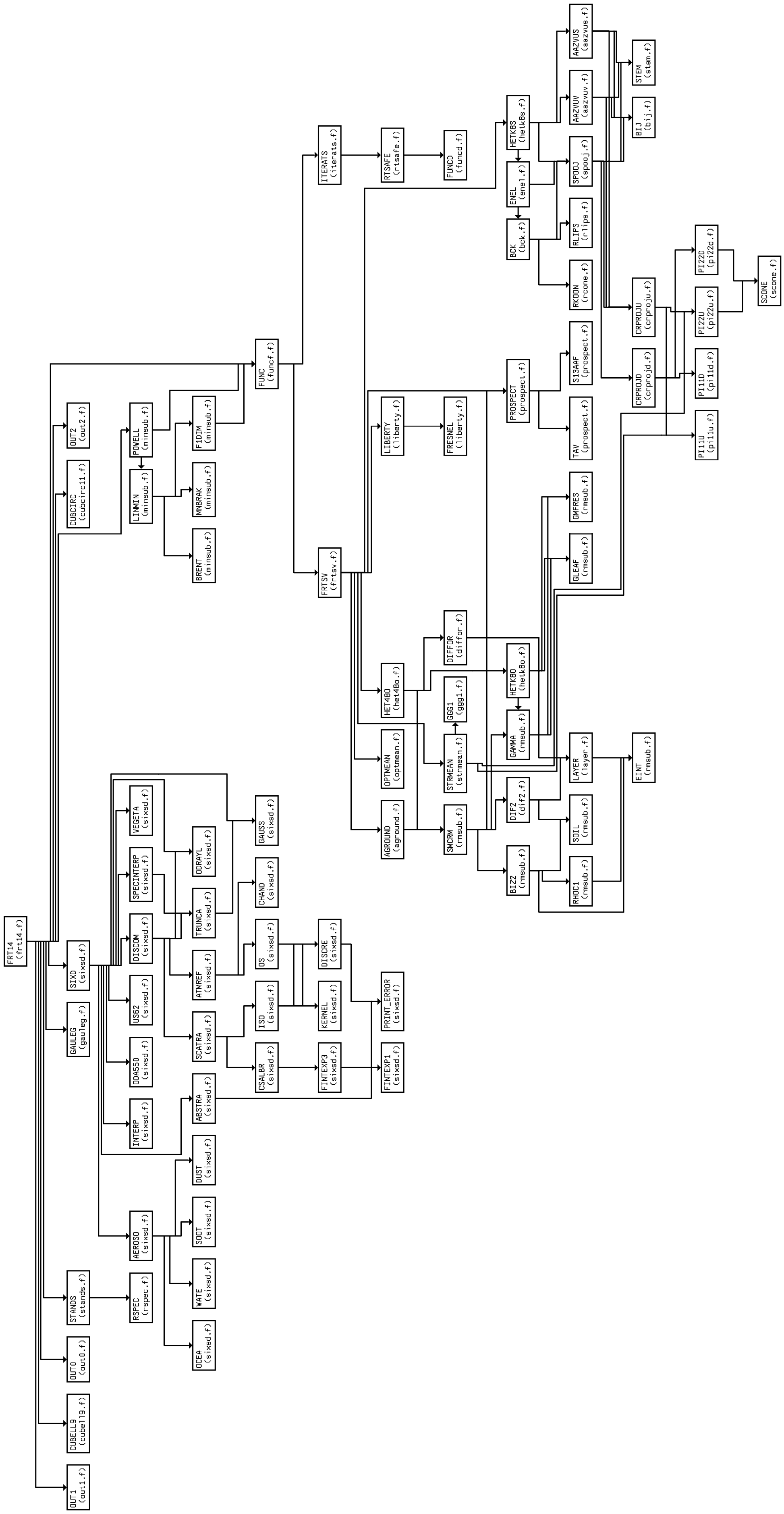

Figure 2: The call-tree of the computer code.

B The usage

The model is distributed as a compressed tar-archive of source texts, sample input and output files. It is recommended to create a separate directory for the model. Move the archive frt14??????.tar.gz to this directory, extract the files and make

```
tar -xzvf frt14??????.tar.gz
make frt14    or    make all
```

make clean   removes object files,
make distclean   removes object files and executables.

If you don't use the gfortran compiler then you should modify the makefile.

To run the code type on the commandline

```
./frt14 inputfile outputfile
```

If you do not give input and output files on commandline then you will be asked for the filenames.

Program frt14 calculates in direct mode forest reflectance and transmittance. There are options to perform calculations in various modes:

- a single run for given Sun and view angles and fixed wavelength
- reflectance spectrum for given view and Sun angles in the given range of wavelengths or for a list of spectral bands
- angular distribution of reflectance at given azimuth (relative to the Sun azimuth) for a given Sun zenith angle in the range of view polar angles 0 .. 80°

Any view and Sun angle is allowed, however, do not use polar angles very close to 90°.

There are several input files required: a file of stand parameters (the stand file), the files of tree parameters for the second, third *etc* tree classes, files of absorption spectra for the leaf optics model, the files of refraction index spectrum and of Price' vectors, and files of bark and trunk reflectance spectra.

The same code is used for the inversion: parameters of the first tree class and/or ground vegetation can be estimated.

B.1 The stand file

The same stand file can be used both for the direct and inverse modes. Some input parameters are needed only in the inverse mode. These parameters are ignored in the direct mode and thus can be missing in the direct mode. The files of the second and other tree classes have the same structure as the stand file for the direct mode, the redundant data may be missing, in case they are present they are not used.

The input parameter $ijob$ controls which task will be run:

ijob	task
0	a single run, Sun and view angles, and wavelength fixed to the first value of the respective parameter in the input file
1	calculate spectrum, Sun and view angles fixed
2	calculate angular distribution for theta = -80 .. 80°, Sun zenith, azimuth and wavelength fixed
3	inversion of the model using BRF values in the fixed view direction, the initial guess, the recommended range of parameters, and errors of the reflectance values are accounted for in the merit function; BRF values at different wavelengths are used
4	inversion of the model, absolute differences in the merit function

ijob = 1
The spectral range is determined by the wavelength of the first spectral band, the wavelength increment dwl, and the number of spectral bands. The valid range of wavelengths is 400 - 2400 nm, spectral resolution 1 nm. The spectrum step is given by an input parameter dwl, if $dwl \leq 0$ then the list of wavelengths should be given.

ijob = 2
Program calculates the angular distribution of forest reflectance and transmittance in the range -80 .. 80° at a given azimuth (relative to the principal plane) and given increment in the view nadir angle. Negative polar angles correspond to the backscattering (hot-spot side), and positive polar angles - to the forward scattering.

ijob = 3
The code is run in inverse mode, n parameters of the first tree class which are listed in the key vector $ll(n)$ are estimated by minimizing the merit function $F(X)$, Eq. (23).

ijob = 4
As $ijob = 3$, except absolute differences are accounted for in the merit function $F(X)$, i.e. $\epsilon = 1$ in Eq. (23).

Structure of the stand file

A sample stand file is printed in the page 20. Colons are used to mark comments, information after a colon is not used by the computer code. Below the sample stand file is commented linewise. The row of the input file is printed in bold. As the number of lines is not constant - it depends on the number of leaf components - the lines in comments are not numbered. The numbers in the right column are the position of the respective parameter in the vector of model parameters.

The first group of parameters describe the task and situation. The second group is the description of the first tree class. The third group is the description of the ground vegetation. In case of inversion the fourt group provides inversion parameters, and the fifth group the input reflectance data.

In the files of other tree classes the lines of the first group are scrolled forward, tree parameters are in the same format as in the file of the first tree class, the other groups of parameters may be missing.

A sample stand file

'RAMI KS' : data set name
49 : stand age
7 : # of size classes
*** files of refractive index and other tree classes ***
'refrind.dat' 'algl1' 'potr1' 'tico2' 'bepe2' 'algl2' 'piab3'
0 : *ijob*: 0-single, 1-spectrum, 2-ad, 3,4-inversion (3-relat., 4-abs. differences)
4 .80 .17 .0 .03 : iaer, c(i) - aerosol data (6S)
0. .06 : v, tau_aer(550) - visibility (6S)
37.6 : Sun zenith
0. 2. 7. : view nadir angle, its increment, and view azimuth angle
6 -1. : # of spectral bands and spectrum step
486. 571. 650. 838. 1677. 2217. : spectral bands (TM)

x0	xmin	xmax	dx		i
'KS'				: species	
t_elli				: crown form	
.0399	.0001	.08	.02	: stand density, m^{-2}	1
26.5	10.	30.	5.	: tree height, m	2
9.0	.5	10.	9.	: crown l, m; ell | con	3
0.	.5	10.	1.	: cylinder	4
1.7	.2	5.	.3	: crown radius, m	5
20.7	2.	25.	5.	: trunk diameter, cm	6
3.014	1.1	5.	8.	: m - total dry leaf weight, kg/tree	7
76.	30.	120.	60.	: SLW - leaf weight per area, g m-2	8
0.	.0	4.5	.5	: eln3 - -ln(1 - eps)	9
53.57	0.	90.	20.	: thm3 - modal leaf angle	10
.2	.05	.6	.2	: shoot length, m	11
0.15	.01	1.	.05	: BAI/LAI	12
1.3	.6	2.8	.05	: tree distr. param. GI_j	13
.0	.0	.6	.05	: H-G asymmetry (phase function)	14
.95	.6	1.	.2	: shoot shading coef	15
.9	.6	1.2	.2	: refr. index ratio	16
1.658	1.2	2.8	.5	: leaf str. param. - PROSPECT N	17
40.	20.0	60.	5.	: d_cell Liberty	18
0.03	.01	0.06	.02	: a_cell Liberty	19
'birchbr1.dat'				: file of branch reflectance	
'birchtr1.dat'				: file of trunk reflectance	
'prospect'				: leaf optics model	
3				: # of leaf components	
144.0	50.	320.	50.	'waterb.dat' : c1, % of SLW, component 1	20
0.60	.3	1.	.2	'chlorp3.dat' : c2, % of SLW, component 2	21
54.65	34.	99.8	20.	'drymatter.dat' : c3, % of SLW, component 3	22
*** Ground vegetation ***					
1.61	.01	6.	.3	: LAI2_ground, upper layer	30
.15	.02	.4	.05	: sl2 - HS-parameter	31
1.0	.4	1.	.2	: clmp2 - foliage clumping parameter	32
0.60	0.	2.	.3	: szz - vertical regularity	33
3.99	.0	4.5	.5	: eln2 - -ln(1 - eps)	34
57.34	0.	90.	20.	: thm2 - modal leaf angle	35
.90	.6	1.3	.2	: n_ratio2	36
76.0	60.	120.	30.	: SLW2(g/m^2)	37
1.315	1.	2.8	.2	: N2 (PROSPECT)	38
'prospect'				: leaf optics model, upper layer	
4				: # of leaf components	
5.0	3.	30.	10.	'waterb.dat' : c1, % of SLW, component 1	39
.633	.3	.8	.2	'chlorp3.dat' : c2, % of SLW, component 2	40
17.60	0.	40.	2.	'anthocyanin.dat' : c3, % of SLW, component 3	41
81.80	74.	99.8	20.	'drymatter.dat' : c4, % of SLW, component 4	42
0.53	.01	1.1	.3	: LAI1_ground, lower layer	49
.15	.02	.4	.05	: sl1 - HS-parameter	50
0.6	.4	1.	.2	: clmp1 - foliage clumping parameter	51
3.0	.0	4.5	.5	: eln1 - -ln(1 - eps)	52
90.	0.	90.	20.	: thm1 - modal leaf angle	53

0.9	.6	1.3	.2	: n_ratio1	54
65.29	60.	120.	30.	: SLW1(g/m^2)	55
1.0053	1.	2.5	.2	: N1 (PROSPECT)	56
’prospect’				: leaf optics model, lower layer	
5				: # of leaf components	
85.23	60.	120.	50.	’waterb.dat’ : c1, % of SLW, component 1	57
.40	.3	.8	.2	’chlorp3.dat’ : c2, % of SLW, component 2	58
0.44	.3	.8	.2	’anthocyanins.dat’ : c3, % of SLW, component 3	59
98.72	94.	99.8	20.	’drymatter.dat’ : c4, % of SLW, component 4	60
.44	.0002	4.	.1	’cellp3.dat’ : c5, % of SLW, component 5	61
’soil.dat’		45.		: file of Price’ vectors, th*	
1.13	0.95	1.4	.1	: s1 - soil parameters	67
.0	-.1	.1	.02	: s2	68
.0	-.05	.05	.02	: s3	69
.0	-.04	.04	.02	: s4	70

* !!! the following linesi are not required for direct problem !!! ***

’powell’				: name of the optimization subroutine
5000	1	100	100	: nfmax, itmax, itbr, nbrak
1.E-9	1.E-7	1.E-13	1.E-8	: zeps, tolbr, tiny, ftolp
1.	.5	2.	.2	: alpha, beta, gamma, dx
2	20.			: n, at
1	7			: ll(i)
486.	.0271	.02		: th_Sun=38.
572.	.2744	.1		: th_Sun=38.
661.	.2806	.1		: th_Sun=38.
838.	.0228	.02		: th_Sun=38.
1677.	.2702	.1		: th_Sun=38.
2217.	.2765	.1		: th_Sun=38.

****

lambda reflectance delta_rho

’RAMI KS’ : data set name
49 : stand age
7 : # of size classes

The number of tree classes, the max number of tree classes is 10.

***** files of refractive index and other tree classes ***** – a comment line

’refrind.dat’ ’algl1’ ’potr1’ ’tico2’ ’bepe2’ ’algl2’ ’piab3’

This line cannot be omitted in the case of one tree class.

1 : *ijob*: 0-single, 1-spectrum, 2-ad,
3,4-inversion (3-relat., 4-abs. differences)

The job control parameter $ijob$:
0 - calculate a single value of canopy reflectance
1 - calculate reflectance spectrum for the given Sun and view angles
2 - calculate reflectance angular distribution at given azimuth
3 - inversion of the FRT model, relative differences in the merit function
4 - inversion of the FRT model, absolute differences in the merit function

The next group of parameters are the input parameters of the 6S model (Vermote et al., 1997).

4 **.80 .17 0. .03** : iaer, c(i) - aerosol data (6S)

$iaer, c(i)$ – aerosol model (6S)
-1 BRDF, no sky radiation
0 no aerosols
1 continental model
2 maritime model
3 urban model
4 enter the volumic percentage of each component $c(i)$
$c(1)$ – fraction of dust-like
$c(2)$ – water-soluble
$c(3)$ – oceanic
$c(4)$ – soot

0. **.06** : visibility v, km, and/or tau_aerosol(550 nm) if v<= 0
37.6 : Sun zenith

0. **2.** **7.** : view nadir angle, its increment, and azimuth angle.
The azimuth angle is counted from the principal plane, the allowed range is [0..180].
6 **-1.** : # of spectral bands, spectrum step

Number of spectral bands; the spectrum step $d\lambda$. If $d\lambda < 0$ then give the list of spectral bands on the next line. Otherway, the spectrum has the fixed increment and only the first wavelength is read.

486. **571.** **650.** ... : spectral bands
x0 **xmin** **xmax** **dx** **i** - a comment
'KS' : tree species, a character string for information purposes only
t_elli : crown form,
A logical parameter of crown shape: t – ellipsoid, f – cylinder+cone

Starting from the next row there are four parameter values in each line. Only the first value (*x0*) is required for the direct run, x_min and x_max are the boundary values of the parameter in the inversion run. The fourth column, dx, is the tolerance of the parameter in the inversion, Eq. (23). The first value (*x0*) serves as an initial guess and as an expert estimate $x_{e,j}$, Eq. (23) of the parameter value in the inversion. There is the parameter number in the vector of parameters in the last column. Only the first column (*x0*) is needed in the direct mode ($ijob$ = 0, 1, 2)

.0399 **.0001** **.08** **.08** : stand density, m^{-2}
Number of trees for the given tree class
26.5 **10.** **25.** **5.** : tree height, m
9.0 **.5** **10.** **9.** : crown l, m; ell | con
Crown length (ellipsoid) or length of the conical part of the crown (cylinder+cone)
0. **0.** **10.** **1.** : cylinder
Length of the cylindrical part of crown
1.7 **.2** **5.** **.3** : crown radius, m

Crown radius - the horizontal semiaxis of ellipsoid or the base radius of the cone

20.7 **2.** **25.** **5.** : trunk diameter, cm

DBH – trunk diameter at the breast height.

3.014 **1.1** **3.** **8.** : m - total dry leaf weight, kg/tree
76. **30.** **120.** **60.** : SLW - leaf weight per area, g m-2
0. **.0** **4.5** **.5** : eln3 - -ln(1 - eps)

the eccentricity parameter of LAD

53.57 **0.** **90.** **20.** : thm3 - modal leaf angle
.2 **.05** **.6** **.2** : shoot length, m
.15 **.01** **1.** **.05** : BAI/LAI ratio
1.48 **.6** **2.8** **.05** : tree distr. param. GI_j

Grouping index, $GI_j = 1$ – a random stand, $GI_j < 1$ – a clumped stand, $GI_j > 1$ – a regular stand.

.0 **.1** **.6** **.05** : H-G asymmetry (phase function)

If this parameter is ≤ 0, then the Ross-Nilson area scattering phase function of plate medium is used.

.95 **.6** **1.** **.2** : shoot shading coef

Shoot shading parameter κ, accounts for the decrease of effective G-function due the mutual shading of leaves (needles) , $\kappa = 1$ – no mutual shading.

1. **.6** **1.2** **.2** : refr. ind. ratio
1.6016 **1.6** **2.8** **.5** : leaf str. param. - - PROSPECT N

Refraction index of the leaf surface wax is calculated from the tabulated value by multiplying to this coefficient.

40. **20.** **60.** **5.** : d_cell Liberty
0.03 **0.01** **0.06** **.02** : a_cell Liberty
'birchbr1.dat' : file of branch reflectance
'birchtr1.dat' : file of trunk reflectance
'prospect' : leaf optics model, options are 'prospect' and 'liberty'.
3 : # of leaf components n_{comp}

In the next n_{comp} lines the percent concentration of the component and the file name of the component absorption spectrum for every component is listed. Despite in the direct mode only the first parameter $x(0)$ is used, the filename must be at the fifth position in the line. The components 20-29 of the vector of parameters are reserved for the leaf biochemical constituents - the tree layer, components 39-48 - the upper layer of ground vegetation, and components 57-66 - the lower layer of ground vegetation, so the maximum number of leaf biochemical components is 10. The components 18 and 19 of the vector of parameters are the LIBERTY parameters cell diameter and amount of inter-cell air, for the tree layer. In ground vegetation only the Prospect model is accepted.

144. **50.** **320.** **50.** **'waterb.dat'** : c1, % of SLW, model component 1
0.60 **.3** **1.** **.2** **'chlorp3.dat'** : c2, % of SLW, model component 2
54.65 **40.** **99.9** **20.** **'drymatter.dat'** : c3, % of SLW, model component 3

The next group of parameters are the input parameters of the two-layer CR model (Kuusk, 2001).

****** Ground vegetation *****			- a comment	
1.61	**.01**	**6.**	**6.**	: LAI2_ground, upper layer
.15	**.02**	**.4**	**.4**	: sl2 - HS-parameter
1.0	**.4**	**1.**	**.2**	: clmp2 - foliage clumping parameter
1.2	**0.**	**2.**	**.3**	: szz - vertical regularity
3.99	**.0**	**4.5**	**.5**	: eln2 - -ln(1 - eps)
53.34	**0.**	**90.**	**20.**	: thm2 - modal leaf angle
.9	**.6**	**1.3**	**.2**	: n_ratio2
76.0	**60.**	**120.**	**30.**	: SLW2(g/m^2)
1.315	**1.**	**2.8**	**.2**	: N2 (PROSPECT)
’prospect’				: leaf optics model, upper layer
4				: # of leaf components
5.	**1.**	**120.**	**50.**	**’waterb.dat’** : c1, % of SLW, component 1
.633	**.3**	**.8**	**.2**	**’chlorp3.dat’** : c2, % of SLW, component 2
17.60	**0.**	**40.**	**20.**	**’anthocyanins.dat’** : c3, % of SLW, component 3
81.80	**60.**	**99.8**	**20.**	**’drymatter.dat’** : c4, % of SLW, component 4
0.53	**.01**	**1.**	**.3**	: LAI1_ground, lower layer
.15	**.02**	**.4**	**.05**	: sl1 - HS-parameter
0.6	**.4**	**1.**	**.2**	: clmp1 - foliage clumping parameter
3.0	**.0**	**4.5**	**.5**	: eln1 - -ln(1 - eps)
90.	**0.**	**90.**	**20.**	: thm1 - modal leaf angle
.9	**.6**	**1.3**	**.2**	: n_ratio1
65.29	**60.**	**120.**	**30.**	: SLW1(g/m^2)
1.0053	**1.**	**2.5**	**.2**	: N1 (PROSPECT)
’prospect’				: leaf optics model, lower layer
5				: # of leaf components
85.23	**60.**	**120.**	**50.**	**’waterb.dat’** : c1, % of SLW, component 1
.4	**.3**	**.8**	**.2**	**’chlorp3.dat’** : c2, % of SLW, component 2
0.44	**.3**	**.8**	**.2**	**’anthocyanins.dat’** : c3, % of SLW, component 3
98.72	**94.**	**99.8**	**20.**	**’drymatter.dat’** : c4, % of SLW, component 4
.44	**.0002**	**4.**	**.1**	**’cellp3.dat’** : c5, % of SLW, component 5
’soil.dat’	**45.**			: file of Price’ vectors, th*
1.1309	**.05**	**.4**	**.07**	: s1 - soil parameters
.0	**-.1**	**.1**	**.02**	: s2
.0	**-.05**	**.05**	**.02**	: s3
.0	**-.04**	**.04**	**.02**	: s4

The next group of parameters are optimization parameters. The only working option for the optimization subroutine is ’powell’.

’powell’				: name of the optimization subroutine
5000	**1**	**100**	**100**	: nfmax, itmax, itbr, nbrak

$nfmax$ – the max number of calculations of merit function
$itmax$ – the max number of iterations
$itbr$ – the max number of iterations in the subroutine brent
$nbrak$ – number of iterations in the subroutine mnbracket

1.E-9	**1.E-7**	**1.E-13**	**1.E-8**	: zeps, tolbr, tiny, ftolp
1.	**.5**	**2.**	**.2**	: alpha, beta, gamma, dx

2	**10.**		: n, at

n - the number of model parameters subject to inversion
at - penalty – the weight w_i, Eq. (23), at = 10. is ok!

7	**1**		: ll(i)

The key vector $ll(n)$, here the ordinal numbers of free model parameters which are subject to estimation are listed.

The next lines are the reflectance values for inversions $ijob = 3$ and $ijob = 4$: for every spectral channel. The number of reflectance/transmittance values should be n_chnl.

486.	**.0271**	**.02**	: th_Sun=38.
572.	**.2744**	**.1**	: th_Sun=38.
661.	**.2806**	**.1**	: th_Sun=38.
838.	**.0228**	**.02**	: th_Sun=38.
1677.	**.2702**	**.1**	: th_Sun=38.
2217.	**.2765**	**.1**	: th_Sun=38.

B.2 A sample file of the second tree class

```
'RAMI KS, pine'                         : data set name
49                                      : stand age
7                                       : # of size classes
*** files of refractive index and other tree classes ***
'refrind.dat' 'algl1' 'potr1' 'tico2' 'bepe2' 'algl2' 'piab3'
0          : *ijob*: 0-single, 1-spectrum, 2-ad, 3,4-inversion (3-relat., 4-abs. differences)
4          .80   .17   .0   .03   : iaer, c(i) - aerosol data (6S)
0.         .06                          : v, tau_aer(550) - visibility (6S)
37.6                                    : Sun zenith
0.         2.        7.                 : view nadir angle, its increment, and view azimuth angle
6          -1.                    : # of spectral bands and spectrum step
486. 571. 650. 838. 1677. 2217.         : spectral bands (TM)
x0         xmin      xmax      dx                                              i
'MA'                                    : species
t_elli                                  : crown form
.0099      .0001     .08       .02      : stand density, m^-2                  1
20.5       10.       30.       5.       : tree height, m                       2
4.0        .5        10.       9.       : crown l, m; ell | con                3
0.         .5        10.       1.       :            cylinder                  4
1.7        .2        5.        .3       : crown radius, m                      5
20.7       2.        25.       5.       : trunk diameter, cm                   6
1.014      1.1       5.        8.       : m - total dry leaf weight, kg/tree   7
120.       30.       120.      60.      : SLW - leaf weight per area, g m-2    8
0.         .0        4.5       .5       : eln3 - -ln(1 - eps)                  9
53.57      0.        90.       20.      : thm3 - modal leaf angle              10
.2         .05       .6        .2       : shoot length, m                      11
0.15       .01       1.        .05      : BAI/LAI                              12
1.3        .6        2.8       .05      : tree distr. param. GI_j              13
.0         .0        .6        .05      : H-G asymmetry (phase function)       14
.95        .6        1.        .2       : shoot shading coef                   15
.9         .6        1.2       .2       : refr. index ratio                    16
1.8        1.2       2.8       .5       : leaf str. param. - PROSPECT N        17
40.        20.0      60.       5.       : d_cell Liberty                       18
0.03       .01       0.06      .02      : a_cell Liberty                       19
'pinebr1.dat'                           : file of branch reflectance
'pinetr1.dat'                           : file of trunk reflectance
```

’prospect’				: leaf optics model	
3				: # of leaf components	
144.0	50.	320.	50.	’waterb.dat’ : c1, % of SLW, component 1	20
0.60	.3	1.	.2	’chlorp3.dat’ : c2, % of SLW, component 2	21
54.65	34.	99.8	20.	’drymatter.dat’ : c3, % of SLW, component 3	22

B.3 Bark and trunk reflectance spectra

The files of bark and trunk reflectance spectra are simple two-column files of 2001 rows, where the first column is wavelength, nm, and the second column is reflectance. The wavelength interval is 1 nm.

C A sample output file

```
#
#   Forest Reflectance Model FRT14 V.04.2025  by A. Kuusk, T. Nilson
#
# Input parameters:
# RAMI KS                                    Stand Age  =   49
##  ijob =   2
#   Sun zenith =  36.0 View azimuth =   0.0  View zenith step =   2.0
#   Wavelength = 671.9 nm
#   7 tree class(es)
#   Files of parameters of other tree classes:
#                           algl1    potr1    tico2    bepe2    algl2
#                            1        2        3        4        5        6
#                            KS       LM       HB       PN       KS       LM
#                            ellips  ellips  ellips  ellips  ellips  ellips
#   1  stand density, m-2   0.0399  0.0176  0.0079  0.0264  0.0066  0.0020
#   2  tree height, m      26.500  23.400  26.760  20.200  17.900  17.500
#   3  ell. or cone         9.000  15.000   8.220  13.000   5.600   8.500
#   4  cylinder, m          0.000   0.000   0.000   0.000   0.000   0.000
#   5  crown radius, m      1.700   2.107   2.100   2.130   1.100   1.500
#   6  trunk d, cm         20.700  22.400  21.600  14.500  10.500  13.100
#   7  total leaf weight    3.014   2.995   5.768   1.640   0.659   0.770
#   8  leaf weight, g m-2  76.000  77.400  76.200  25.500  76.000  77.400
#   9  eln                  0.000   3.600   5.500   5.700   0.000   3.600
#  10  thm                 53.570   6.800   8.190   6.300  53.570   6.800
#  11  shoot size, m        0.200   0.150   0.200   0.100   0.200   0.150
#  12  BAI/LAI              0.150   0.220   0.100   0.080   0.150   0.220
#  13  tree distr. param.   1.200   1.480   1.480   1.200   1.480   1.480
#  14  g_H-G                0.000   0.000   0.000   0.000   0.000   0.000
#  15  shoot shading coef   0.950   0.950   0.950   0.950   0.950   0.950
#  16  refr. ind. ratio     0.900   0.900   0.900   0.900   0.900   0.900
#  17  leaf str.par         1.658   1.762   1.548   1.543   1.658   1.762
#  18  D_cell, mcm         40.000  40.000  40.000  40.000  40.000  40.000
#  19  i-cell air           0.030   0.030   0.030   0.030   0.030   0.030
# bark refl. files: birchbr1.dat   aldertr1.dat   oambr1.dat   oambr1.dat
# trunk refl. files birchtr1.dat   aldertr1.dat   oamtr1.dat   oamtr1.dat
# Leaf models:     prospect  prospect  prospect  prospect  prospect
#  # of leaf comp-s:        3        4        4        3        3
# 20       waterb.dat  waterb.dat  waterb.dat  waterb.dat  waterb.dat
#          144.0000    147.8000    56.8100     103.2000    144.0000
#  21      chlorp3.dat chlorp3.dat chlorp3.dat chlorp3.dat chlorp3.datat
#            0.6000    0.8772      0.2778      0.2890      0.6000
#  22      drymatter.dat drymatter.dat drymatter.dat drymatter.dat
#          54.6500       71.2900       18.5700       102.9000      54.6500
#  23      _           base.dat    base.dat    _           _           _
#          0.0000      167.5000    20.5300     0.0000      0.0000      0.0000
#
```

```
# *** Ground vegetation, upper layer,       lower layer
#  30  ground LAI2            1.610           0.530
#  31  leaf size              0.150           0.150
#  32  clmp                   1.000           0.600
#  33  szz                    0.600
#  34  eln                    4.000           3.000
#  35  thm                   57.340          90.000
#  36  n_ratio                0.900           0.900
#  37  SLW                   76.000          65.290
#  38  leaf str.par           1.315           1.005
#   Leaf model: prospect
#  39  # of leaf components:      4               5
#  39     waterb.dat          5.000    waterb.dat          85.230
#  40     chlorp3.dat         0.633    chlorp3.dat          0.400
#  41     anthocyanins.dat   17.600    anthocyanins.dat     0.440
#  42     drymatter.dat      81.800    drymatter.dat       98.720
#  43                                  cellp3.dat          0.440
# File of Price' vectors: soil.dat
# Sun angle of the soil reflectance:  45.0
#  67  s1_soil               1.1309
#  68  s2                    0.0000
#  69  s3                    0.0000
#  70  s4                    0.0000
#
#
# *** Results:
#                            1         2         3         4         5
#                            KS        LM        HB        PN        KS
#                            ellips    ellips    ellips    ellips    ellips
#    stand density, m-2      0.040     0.018     0.008     0.026     0.007
#    tree height, m         26.500    23.400    26.760    20.200    17.900
#    ell. or cone            9.000    15.000     8.220    13.000     5.600
#    cylinder, m             0.000     0.000     0.000     0.000     0.000
#    crown radius, m         1.700     2.107     2.100     2.130     1.100
#    trunk d, cm            20.700    22.400    21.600    14.500    10.500
#    lf_wght/tr&tot_m-2      3.014     2.995     5.768     1.640     0.659
#    SLW, g m-2             76.000    77.400    76.200    25.500    76.000
#    eln                     0.000     3.600     5.500     5.700     0.000
#    thm                    53.570     6.800     8.190     6.300    53.570
#    shoot size, m           0.200     0.150     0.200     0.100     0.200
#    BAI/LAI                 0.150     0.220     0.100     0.080     0.150
#    tree distr. param.      1.200     1.480     1.480     1.200     1.480
#    g_H-G                   0.000     0.000     0.000     0.000     0.000
#    shoot shading coef      0.950     0.950     0.950     0.950     0.950
#    refr. ind. ratio        0.900     0.900     0.900     0.900     0.900
#    leaf str.par            1.658     1.762     1.548     1.543     1.658
#    D_cell, mcm            40.000    40.000    40.000    40.000    40.000
#    i-cell air              0.030     0.030     0.030     0.030     0.030
#   bark refl. files:  birchbr1.dat  aldertr1.dat  oambr1.dat  oambr1.dat
#   trunk refl. files  birchtr1.dat  aldertr1.dat  oamtr1.dat  oamtr1.dat
#   Leaf models:       prospect      prospect      prospect    prospect
#  # of leaf comp-s:     3              4              4              3
#                  waterb.dat     waterb.dat     waterb.dat     waterb.dat
#                  144.0000       147.8000        56.8100       103.2000
#                  chlorp3.dat    chlorp3.dat    chlorp3.dat    chlorp3.dat
#                    0.6000         0.8772         0.2778         0.2890
#                  drymatter.dat  drymatter.dat  drymatter.dat  drymatter.dat
#                   54.6500        71.2900        18.5700       102.9000
#                  _              base.dat       base.dat       _
#                    0.0000       167.5000        20.5300         0.0000
#   rl_eff =  0.1179  tl_eff =  0.0831  rsl =  0.1160
#   leaf area density       0.692     0.325     1.047     0.536     0.672
#   Total LAI     4.667
#   Total BAI     0.600
```

```
#   crown closure =  1.150                          canopy closure =  0.749
#
# Warning!: canopy closure (CC) > crown closure (CR)
#           class, CC, CR:   7  0.2095E-01  0.1764E-01
#
#
#  *** Ground vegetation, upper layer,       lower layer
#   ground LAI2, LAI1       1.610           0.530
#   leaf size               0.150           0.150
#   clmp                    1.000           0.600
#   szz                     0.600
#   eln                     4.000           3.000
#   thm                    57.340          90.000
#   n_ratio                 0.900           0.900
#   SLW                    76.000          65.290
#   leaf str.par            1.315           1.005
#   Leaf model: prospect
#   # of leaf components:       4             5
#      waterb.dat           5.000     waterb.dat          85.230
#      chlorp3.dat          0.633     chlorp3.dat          0.400
#      anthocyanins.dat    17.600     anthocyanins.dat     0.440
#      drymatter.dat       81.800     drymatter.dat       98.720
#                                     cellp3.dat           0.440
#   s1_soil                1.1309
#   s2                     0.0000
#   s3                     0.0000
#   s4                     0.0000
#
#   Sun zenith =  36.0   View azimuth =   0.0   View zenith step =   2.0
#   Wavelength = 671.9 nm        S'/Q =   0.9
#   CaCl:   0.7485               CrCl:   1.1503
#
# thv   refl     r_grnd   rcr1     rgr1     rdif     b_down    gfr
 -80.0  0.03343  0.02825  0.02326  0.00000  0.01017  0.04473  0.00013
 -78.0  0.03248  0.02751  0.02258  0.00000  0.00990  0.04541  0.00046
 -76.0  0.03175  0.02703  0.02210  0.00000  0.00964  0.04587  0.00110
 -74.0  0.03120  0.02674  0.02178  0.00001  0.00941  0.04622  0.00213
 -72.0  0.03078  0.02660  0.02158  0.00001  0.00920  0.04651  0.00361
[..]
  70.0  0.02571  0.01776  0.01670  0.00001  0.00900  0.04357  0.00552
  72.0  0.02650  0.01851  0.01730  0.00001  0.00920  0.04365  0.00361
  74.0  0.02746  0.01945  0.01804  0.00000  0.00941  0.04373  0.00213
  76.0  0.02864  0.02062  0.01900  0.00000  0.00964  0.04375  0.00110
  78.0  0.03013  0.02209  0.02023  0.00000  0.00990  0.04362  0.00046
  80.0  0.03204  0.02392  0.02187  0.00000  0.01017  0.04332  0.00013
```

D Description of the subroutines

D.1 Subroutines of general use

D.1.1 Function *func*

Function: In the direct mode the function *func* extracts the model parameters from the vector of parameters and provides to the subroutine *frtsv(..)* for a single direct run.

In the inverse mode the function *func* checks that the model parameters are in the allowed range, scales the parmeters subject to the inversion, and computes the merit function.

D.1.2 Subroutine *stands*

reads input data

D.1.3 Subroutine *out0*

prints parameter values to the output file

D.1.4 Subroutine *out1*

prints the results of the direct run to the output file

D.1.5 Subroutine *out2*

prints the results of inversion to the output file

D.1.6 Subroutine *frtsv*

frtsv(..) is the procedure which performs the single direct run of the model.

D.1.7 Subroutines *iterats, rtsafe* and *funcd*

Function: To compute the Fisher's grouping index GI_j, Eq. (10) from the given structure parameter $c_j(\theta_1)$.

Description: The Newton-Raphson method is used, Press et al. (1992), Algorithm 9.4.

D.1.8 Subroutines *cubell9, cubcirc* and *gauleg*

Function: Provide quadrature (cubature) knots and weights to numerical integrations

D.1.9 Subroutine *rspec*

Function: Reads tabulated spectra – absorption spectra of leaf constituents, stem and branch bark reflectance, Price' vectors *etc.*

D.2 Structure modules

D.2.1 Subroutine *strmean*

Function: Computes the mean values of structure parameters.

D.2.2 Subroutine *regre*

Function: Regressions for tree parameters. The call of this subroutine is commented out. Such regressions can be used in case some tree parameters are not available.

D.2.3 Subroutine *ggg*

Function: The Ross-Nilson G-function for elliptical LAD.

D.2.4 Subroutine *hetk8s*

Function: Coordinates the calculation of free lines of sight in Sun and view directions.

D.2.5 Subroutine *enel*

Function: Integrates the bidirectional probability p_{00j}, over the whole tree crown, Eq. (3), and computes the probability to see the sunlight trunk.

Description: The volume integral $\underset{V_j}{\int\int\int} p_{00j}(x, y, z; r_1, r_2)\, dx\, dy\, dz$ is calculated using a cubature for a sphere (ellipsoid) or cubature for a circle and Gauss-Legendre quadrature in respect of the z-coordinate for a cone+cylinder.

D.2.6 Subroutine *bck3*

Function: Computes the bidirectional gap probability p_{ooj}, Eq. (4).

D.2.7 Subroutine *spooj*

Function: Computes the between-crown gap probability p_2, Eq. (4).

Description: The overlapping of crown projections in Sun and view directions $S_{cj}()$ is calculated so that the crown projections S_1 and S_2 in Sun and view directions, respectively, are substituted with circles of the same area. Centers of the circles are halfway between the projections of the base and the top of a crown, see Fig. 2 (p. 7).

D.2.8 Subroutines *rlips* and *rkoon*

Function: Subroutines *rlips* and *rkoon* compute the distance from the given point $M(x, y, z)$ to the perimeter of the ellipsoid or cone+cylinder, respectively, in the given direction (θ, φ).

D.2.9 Subroutines *pi11u, pi11d, pi22u* and *pi22d*

Function: Subroutines *pi11u, pi11d, pi22u* and *pi22d* compute projections of the crown part above (*pi11u, pi22u*) and below (*pi11d, pi22d*) the given level z.

D.2.10 Subroutine ***scone***

Function: Computes the projection area of a cone/frustum of a cone for a given direction.

D.2.11 Subroutine ***stem***

Function: Computes the projection area of a stem.

D.3 Optics modules

D.3.1 Subroutine ***optmean***

Function: Computes the mean and effective values of optical parameters.

D.3.2 Subroutine ***aground***

Function: Computes the directional-hemispherical reflectance $rsdgrou$ and albedo (hemispherical-hemispherical reflectance) $rddgrou$ of ground vegetation.

Description: The double integral over hemisphere which is needed for the hemispherical-hemispherical reflectance of ground vegetation is substituted by an integral over polar angle at the azimuth $\varphi = 90°$. The integral is calculated with an Gaussian quadrature.

D.3.3 Subroutine ***het48o***

Function: Computes radiances down and up, and transmittance of the tree layer.

D.3.4 Subroutine ***hetk8o***

Function: Sums together radiance of all tree classes.

D.3.5 Subroutine ***diffor***

Function: Computes diffuse fluxes of multiple scattering and of scattered diffuse sky radiation.

Description: Diffuse fluxes are computed in two-stream approximation (Bunnik, 1978; Kuusk, 2001).

D.4 Reflectance of ground vegetation

Subroutines

smcrm
biz2
gamma
gleaf

gmfres
soil
dif2
layer
rhoc1
skylspec
constitute the two-layer homogeneous canopy reflectance model ACRM. The full description of algorithms is published by Kuusk (1994, 1995a,b, 2001).

D.5 PROSPECT - the leaf optics model

Subroutines
prospect
tav
s13aaf
constitute the leaf optics model by Jacquemoud and Baret (1990).

D.6 LIBERTY - the leaf optics model

Subroutines
liberty
fresnel
constitute the leaf optics model by Dawson et al. (1998).

D.7 Atmosphere radiative transfer model 6S

General description of the 6S model is published by Vermote et al. (1997). The detail description of 6S modules is in (Vermote et al., 1994). For the calculation of incoming fluxes are used the modules
sixd
abstra
aeroso
atmref
chand
csalbr
discom
discre
dust
gauss
interp
iso
kernel
ocea
oda550
odrayl
os

print_error
scatra
soot
specinterp
trunca
us62
vegeta
wate

D.8 Optimization modules

The Powell's method (Press et al., 1992), Algorithm 10.5 is used for the minimization of the merit function Eq. (16). The corresponding subroutines are
powell
linmin
mnbrak
function brent

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